

37.4: Filter Design for Void-and-Cluster Dither Arrays

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Abstract: The Void-and-Cluster method for producing ordered dither arrays is powerful in its simplicity and flexibility. Central to the method is the void- and cluster-finding filter. This paper explores the use of a gaussian filter, and in particular, the effect of the gaussian parameter σ on image quality. Excellent isotropic blue-noise dithering can be achieved with a σ between 1.0 and 1.5 with relatively small period arrays. Smaller values of σ tend to leave slightly inhomogeneous voids, while larger values tend to form regularly spaced clusters.

INTRODUCTION

The void-and-cluster method for generating dither arrays¹ provides a general spatial-domain means for producing a wide range of ordered-dither arrays. The point process of ordered dither is very attractive in that it is very fast and simple to implement, as storage and processing of neighboring pixels are not required.

An earlier technique for generating ordered dither arrays, called the method of recursive tessellation², built a dither array by placing minority pixels in the center of the largest void consisting of majority pixels, to optimize homogeneity. The very familiar patterns popularized by a paper by Bayer³ are a subset of the resulting recursive tessellation arrays. Building on this algorithm, the void-and-cluster method also builds dither arrays by seeking the biggest voids in which minority pixels can be inserted, but also finds the tightest clusters from which pixels can be removed; this allows the starting point to be anywhere between 0 and 100% gray, as opposed to only those endpoints in the method of recursive tessellation. Key to the nature of the generated dither arrays is the choice of void-finding and cluster-finding filter, which is the focus of this paper.

With an appropriate filter, the void-and-cluster method will produce complete recursive tessellation dither arrays given an initial binary pattern, a starting point, equal to any fixed gray level pattern from recursive tessellation. Such an initial binary pattern can be a solitary minority pixel, or the checkerboard pattern, for example.

As was shown¹, the void-and-cluster method can also produce arrays that generate blue-noise dithering. Blue noise, or high-frequency white-noise, was identified as a very desirable property of dispersed-dot dither patterns⁴. Mista and Parker⁵ have demonstrated a very

interesting means for producing blue-noise dither arrays by direct manipulation of the power spectrum to achieve the desired blue-noise shape; a further refinement is presented in this conference⁶. The void-and-cluster method, like the method of recursive tessellation, fabricates dither arrays entirely in the spatial domain.

THE FILTER

As described earlier¹, the same void- and cluster-finding filter is used to both (1) relax an arbitrary input binary pattern to form an initial binary pattern, or starting point, and (2) build the dither array. Since the dither array for ordered dither is periodic, the corresponding binary pattern will tile all of two-space as shown in Figure 1. The method allows arrays of arbitrary size M by N . This is particularly useful for generating arrays for displays with non-square pixels.

A fundamental feature of a filter that looks for voids or clusters is the wrap-around property as represented by the coverage of the disk outlined in the figure. When the span of such a filter extends beyond the bounds of the binary pattern, it must wrap around to the other side, as indicated by the dashed lines. As with the method of recursive tessellation, this feature is important for avoiding pattern discontinuities at period boundaries.

A void-finding filter considers the neighborhood around every majority pixel in the binary pattern, and a cluster-finding filter considers the neighborhood around every minority pixel. A variety of filters, both linear and nonlinear, symmetric and asymmetric, will work.

It is known that a form of wrap-around linear convolution will work well. In this case, the measure of voidness or clusterness around a candidate location (x,y) within a binary pattern $B(x,y)$ can be expressed

$$\sum_{p=-M/2}^{M/2} \sum_{q=-N/2}^{N/2} B(p',q')f(p,q)$$

where

$$p' = (M + x - p) \bmod M$$

$$q' = (N + y - q) \bmod N$$

and $f(x,y)$ is the filter.

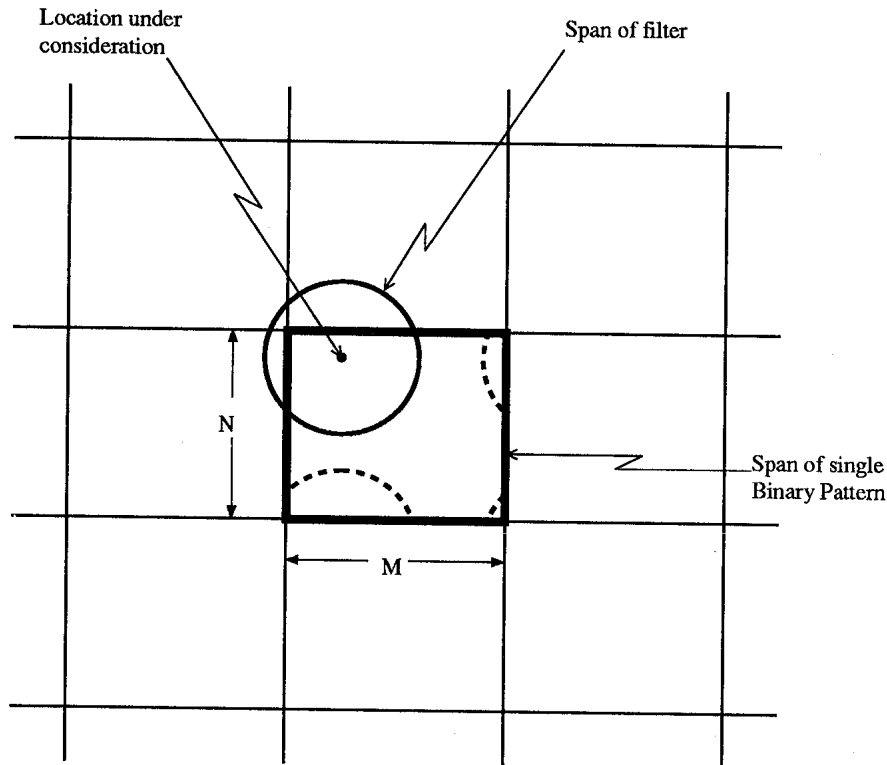


Figure 1. The two-dimensional wrap-around property of the void- and cluster-finding filter. The N by M matrix tiles all of 2-space.

THE EFFECT OF FILTER WIDTH

While assessment of the quality of a generated dither array requires evaluation of the resulting dithered patterns for all gray levels, much can be learned about candidate filters by observing the generation of initial binary patterns. In the following experiment, a symmetric gaussian,

$$f(x, y) = e^{-\frac{x^2+y^2}{2s^2}},$$

is used as the void- and cluster-finding filter, where we examine the effect of changing the value of σ in units of pixel period.

Figure 2 shows the result of using this filter on a white noise input image to produce initial binary patterns for use in generating dither arrays. In all cases a 32 by 32 pattern is used. Four periods are shown (2 vertically and 2 horizontally) to show the effect of tiling, and to reveal any edge discontinuities. We use as an input a white noise pattern with 13% black pixels, as shown in (a). An initial binary pattern is formed from

this input by relaxation process, where the black pixel from the tightest cluster is moved to the center of the biggest void. The process nicely converges when the removal of a pixel from the tightest cluster forms the biggest void¹.

A gaussian filter with $\sigma=0.2$ is used to relax the input pattern to that shown in Figure 2(b). The process is repeated for (c) $\sigma=1.0$, (d) $\sigma=1.5$, (e) $\sigma=2.0$, (f) $\sigma=3.0$, (g) $\sigma=4.0$, and (h) $\sigma=10.0$. Excellent isotropic blue-noise patterns result for σ in the range 1.0 to 1.5, even for this relatively small (32 by 32) array. It is important to note that this judgment is made by observing the effect of dithering all gray levels with the resulting dither array. For values less than 1.0, the output patterns tend to have disproportionately large voids. For larger values, clusters tend to form.

A very curious result occurs for σ greater than 4.0 -- homogeneously distributed groupings form. For filters with such extreme spans, our void- and cluster-finding criterion is somewhat changed. The number of groupings is not effected by gray level, or by filter span, once over the grouping threshold; it only appears to be

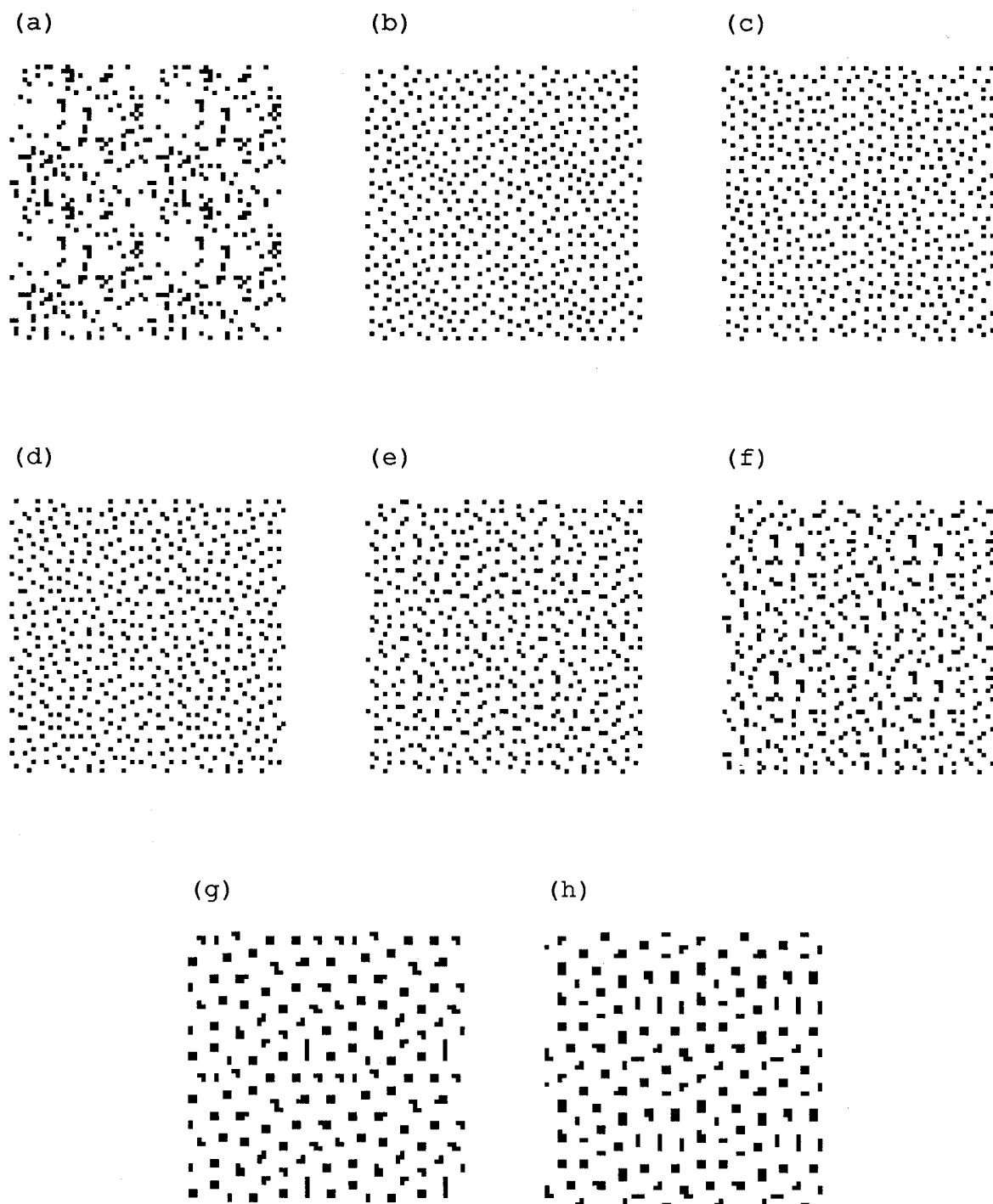


Figure 2. Effect of relaxing an 87% gray random input pattern (a) with a void- and cluster-finding gaussian filter of various widths, σ , in terms of pixel spacing. Initial binary patterns are formed with (b) $\sigma=0.2$, (c) $\sigma=1.0$, (d) $\sigma=1.5$, (e) $\sigma=2.0$, (f) $\sigma=3.0$, (g) $\sigma=4.0$, and (h) $\sigma=5.0$. Note that in all cases, four periods of each 32 by 32 pattern are shown to illustrate the wrap-around properties at the edges of the repeating tile.

dependent on the size of the array. However, as gray level approaches 50%, the groupings start to merge in a herringbone fashion, unlike conventional clustered-dot screening. Perhaps another filter choice could produce a useful moiré-inhibiting cluster-dot screen, where the cluster centers have a blue-noise spectrum.

REFERENCES

¹R. Ulichney, "The void-and-cluster method for dither array generation," *Proc. SPIE, Human Vision, Visual Processing, and Digital Display IV*, vol. 1913, pp. 332-343, Feb. 1-4, 1993.

²R. Ulichney, *Digital Halftoning*, The MIT Press, Cambridge, chapter 6, 1987.

³B.E. Bayer, "An optimum method for two level rendition of continuous-tone pictures," *Proc. IEEE Int. Conf. Commun., Conference Record*, pp. (26-11)-(26-15), 1973.

⁴R. Ulichney, "Dithering with blue noise," *Proc. IEEE*, vol. 76, no. 1, pp. 56-79, 1988.

⁵T. Mitsa and K.J. Parker, "Digital halftoning technique using a blue-noise-mask," *J. Opt. Soc. Am. A*, vol. 9, pp. 1920-1929, 1992.

⁶M. Yao and K.J. Parker, "A comparison of the blue noise mask and error diffusion," *This conference*, 1994.