

# Quantifying Performance of Overlapped Displays

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## ABSTRACT

We consider two physical systems where overlapped displays are employed: (1) Wobulation –a single projector that rapidly shifts the entire display in time by a subpixel amount; (2) Several projector displays overlaid in space with a complex array of space-varying subpixel offsets. In this work we focus on quantifying the resolution increase of these approaches over that of a single projector. Because of the nature of overlapping projections with different degrees of perspective distortion, overlaid pixels have space-varying offsets in both dimensions. Our simulator employs the perspective transformation or homography associated with the particular projector geometry for each subframe. The resulting simulated displays are stunningly accurate. We use “grill” patterns to assess the resolution performance that vary in period, phase, and orientation. A new Fourier-based test procedure is introduced that generates repeatable results that eliminate problems due to phase and spatial variation. We report on results for 2 and 4 position wobulation, and for 1, 2, 4, and 10 overlaid projectors using the frequency-domain based contrast modulation metric. The effects of subpixel phase are illustrated for various grill periods. The results clearly show that resolution performance is indeed improved for overlapped displays.

**Keywords:** Overlapped Displays, Resolution, Homographies, Grill patterns

## 1. INTRODUCTION

New display technologies that are the ensemble of two or more overlapping frames call for a unique means for measuring those displays. The solution to this quantification problem is the topic of this paper.

We overlap projected displays in one of two ways: (1) Wobulation<sup>1</sup> –A single projector rapidly shifts the entire display in time by a subpixel amount, with the speed of oscillation that is not perceptible by the visual system; the offsets are uniform and fixed across the entire display. (2) Several projector displays overlaid in space (rather than in time) with a complex array of space-varying subpixel offsets; the configuration we model is one where subframes are generated to be optimal using an MSE criterion<sup>2,3</sup>. Figure 1 illustrates these two approaches for a four frame case where each frame is identified by a unique color.

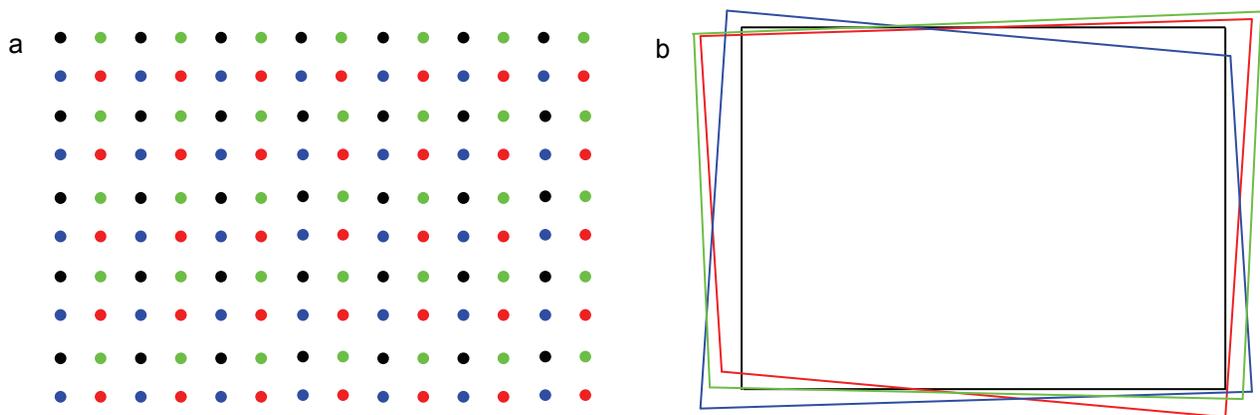


Figure 1. Two means for overlaying projected frames. (a) Single wobulated projector. (b) Several projectors.

Starting with the pixels of a the first frame shown in black in Figure 1(a) successive frames from the same projector are offset rapidly in time by a half pixel to the right, below, and both right and below. In (b) frames from four separate projectors are superimposed and experience different perspective warping because their necessarily different angles to the screen.

These approaches are driven by the fact that the cost of higher resolution displays is much greater than the sum of lower resolution displays. In both cases resolution is increased by skillfully mapping a high resolution image to the subframes. In the case of multiple overlaid projectors, the principal goal is to increase luminous flux in highly cost effective way; improved detail rendition is a bonus.

In Figure 2 a close-up of output from a single projector is compared with a 10-projector system of the same input. It is clear that overlaid displays improve quality; the focus of this paper is the measurement of the improvement.

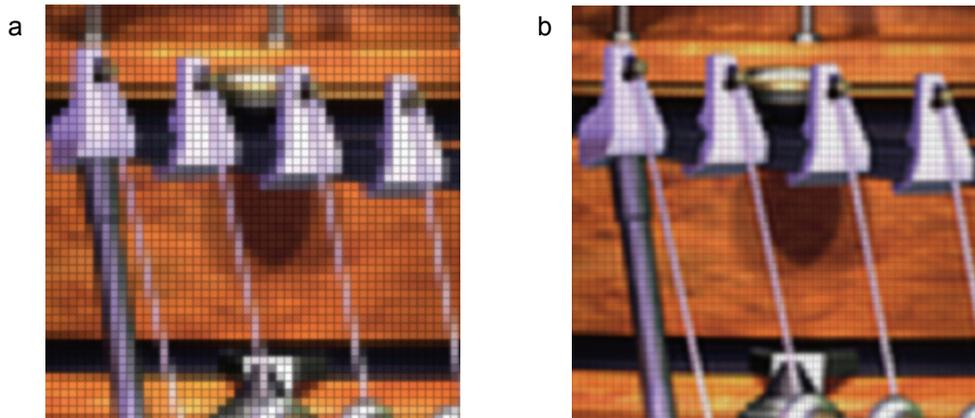


Figure 2. Cropped simulator output for (a) a single projector and (b) 10 overlaid projectors.

Another concern and motivation for this work was the assertion that overlapping projectors cannot improve resolution; it had been argued<sup>4</sup> that aliasing could not be eliminated. To the contrary, it was later theoretically determined<sup>3,5</sup> that aliasing effects of the non-uniform sampling may indeed be cancelled by a proper choice of subframes to produce alias-free super-Nyquist frequencies. In this study we validate this by quantifying contrast modulation for overlaid projections.

Our solution to the measurement problem has two parts: a simulator and a test procedure.

## 2. SOFTWARE SIMULATOR

While Wobulation produces precise and homogenous pixel offsets, the nature of multiple overlapping projections with different degrees of perspective distortion result in overlaid pixels that have space-varying offsets in both dimensions. Any measurement will require averaging over the entire image. While physically projecting such patterns is possible, direct measurement with instrumentation would take an extremely long time. To address this need we designed a detailed software simulator to allow for automated testing. The tool also allows us to explore changes in the projector system that would be otherwise very difficult to implement.

The three main sections of the simulator are shown in Figure 3. The Test Pattern Generator synthesizes a set of grills or patterns of alternating black and white bars of varying period, phase and orientation. The high resolution input image has a size in pixels of  $(X_i, Y_i)$  and can be up to 4096 by 3072 in our implementation.

## 2.1 Subframe Generator

The Subframe Generator is precisely the same as that used for the real projector systems<sup>3</sup>. Our image formation model is a signal processing model that predicts the output given a set of subframes. The model comprises standard signal processing operators such as upsampling, filtering with a pixel point spread function, geometric warping and summation. The Subframes generated can be up to 4 times smaller than the input image size. In our experiments the subframe size ( $X_s, Y_s$ ) was 320 by 240. The number of subframes to be generated,  $N$ , is also specified here. For multiple overlaid projector simulations we used  $N=1, 4$ , and 10. For wobulation  $N=1, 2$ , and 4.

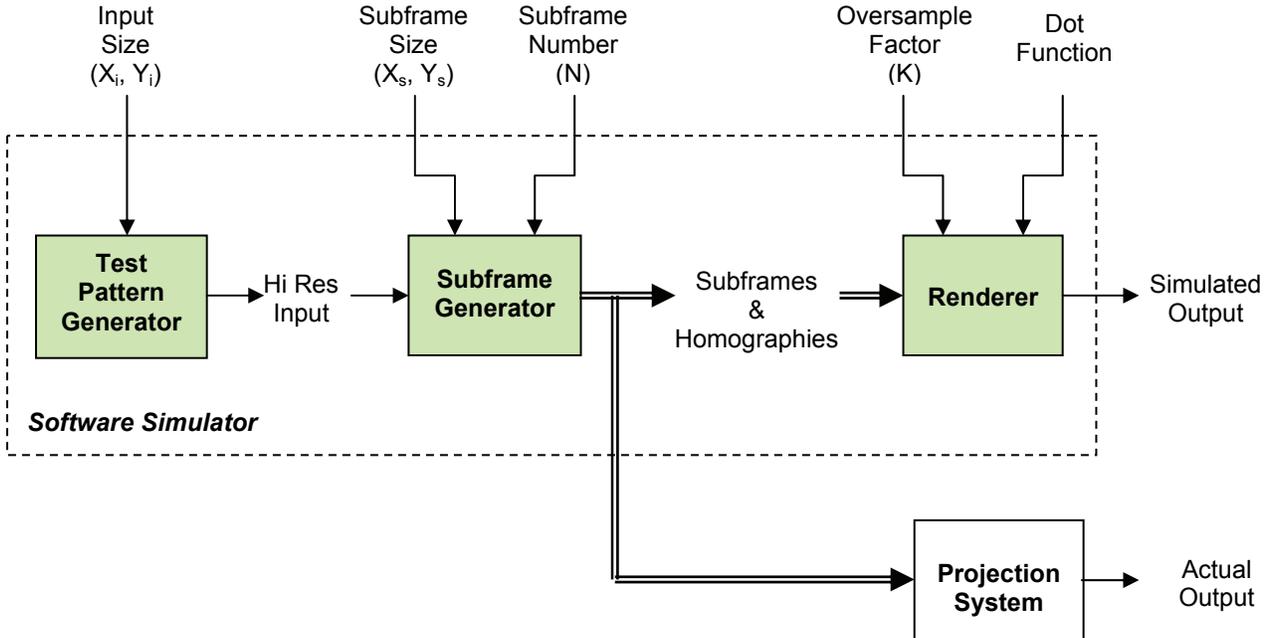


Figure 3. Components of the Software Simulator.

Given the image to be projected, we seek the component sub-frames that maximize the probability that the image predicted by the model is the true hi-resolution image. To make our solution robust to minor calibration errors and noise, we impose a smoothness requirement based on a smooth 2nd derivative. This results in an iterative process that minimizes the mean squared error between the simulated image formed via an image formation model and the true desired hi-resolution image. Intuitively, the process may be understood as computing an error between the given high resolution image (ground truth) and the model's prediction for a given choice of subframes, projecting this error back onto the sub-frames to form better estimates of the sub-frames, and iterating.

## 2.2 Renderer

The resulting subframes are then fed to the Renderer shown in more detail in Figure 4. The Oversample Factor,  $K$ , specifies the number of extra pixels in both  $X$  and  $Y$  that each pixel in a subframe will be expanded to in the simulation. The higher the value of  $K$  the higher the quality of the simulated results. This oversampling is realized by the Expander that effectively places the pixels from each subframe onto a grid that is  $K \times K$  larger with zeros padding the in between locations. A graphic illustration of the rendering of one subframe is show in Figure 5 where the expansion factor  $K = 4$  is used. In our experiments we used a value of  $K = 10$ ; thus a pixel is mapped to a  $10 \times 10$  block where 99 of the pixels are zeros.

Next the actual dot function is used. As current products and product prototypes use DMD projectors, we measured the DMD dot function. Intel's Digital Micromirror Device (DMD) employs tiny mirrors as shown in the micrograph in Figure 6a. Note the hole due to the center post. The blank space between mirrors is imaged as the "screen door" effect.

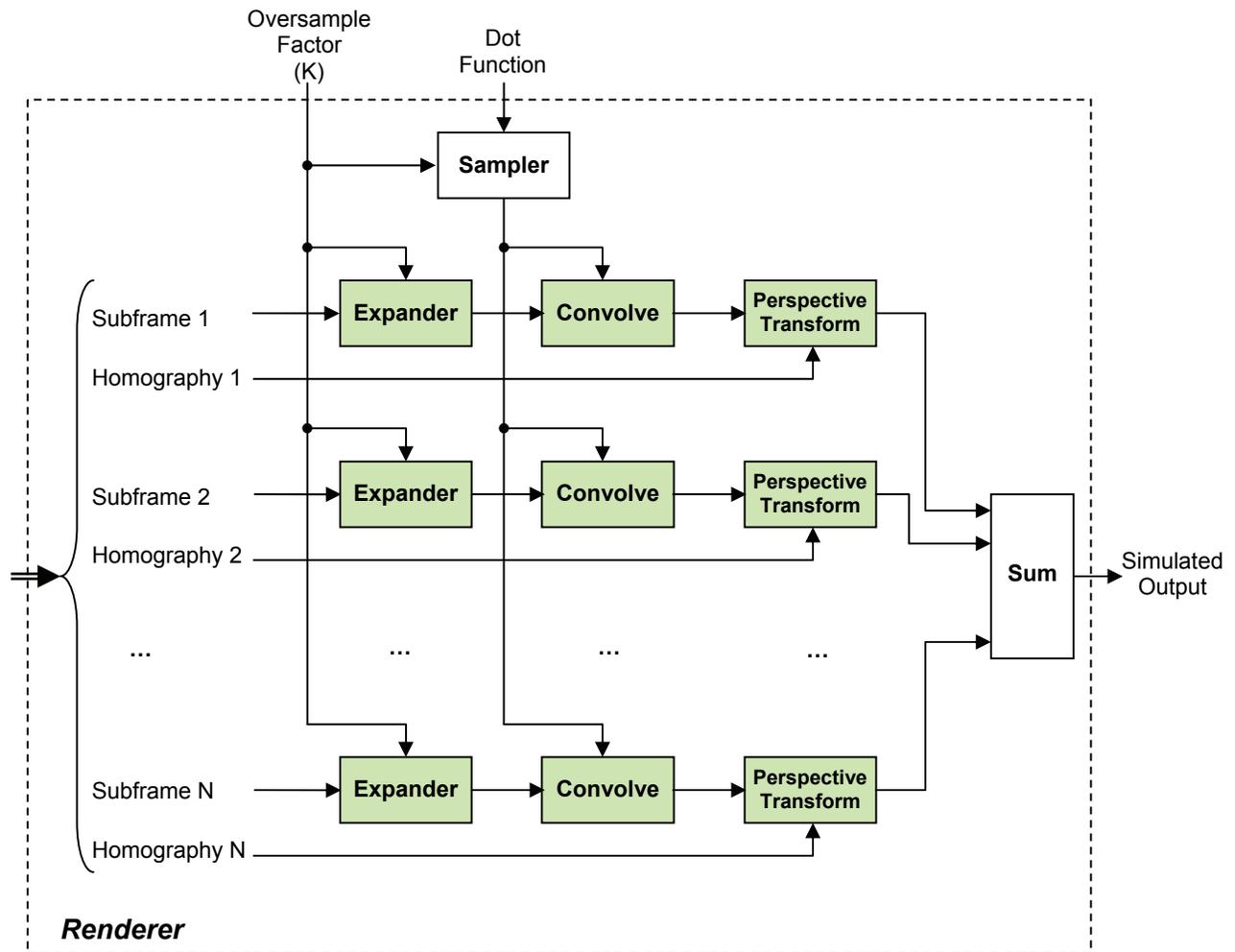


Figure 4. Components of the Renderer.

We projected isolated white pixels on a black background, captured high resolution images of the dot in the center of a 3x3 pixel field, and averaged several together to minimize noise. The dot function used is shown in Figure 6b.

As a test of the validity of modeling output with this dot function we used as input a bitonal image with both isolated white pixels on a black background and clustered white pixels. The output from our simulator along with a macro photo of an actual projection of the same test image on a screen is shown in Figure 7. Convolution with our dot function successfully mimics the gradual tapering of isolated pixels and the more uniform pixels surrounded by the screen door effect in solid areas.

In Figure 4 the dot function is first sampled to be compatible with the oversampled subframes. In our example of  $K=10$ , the dot function would be a 30x30 pixel image. It is then convolved with each of the expanded subframes. The perspective transformation or homography associated with the particular projector geometry for each subframe, represented as a 3x3 linear transformation, warps the result. It is important to apply the perspective transformation at this stage to allow for the much more manageable space-invariant convolution with the dot function. In the case of Wobulation the homography is a simple global shift.

The rendered subframes are summed to produce the final simulated output.

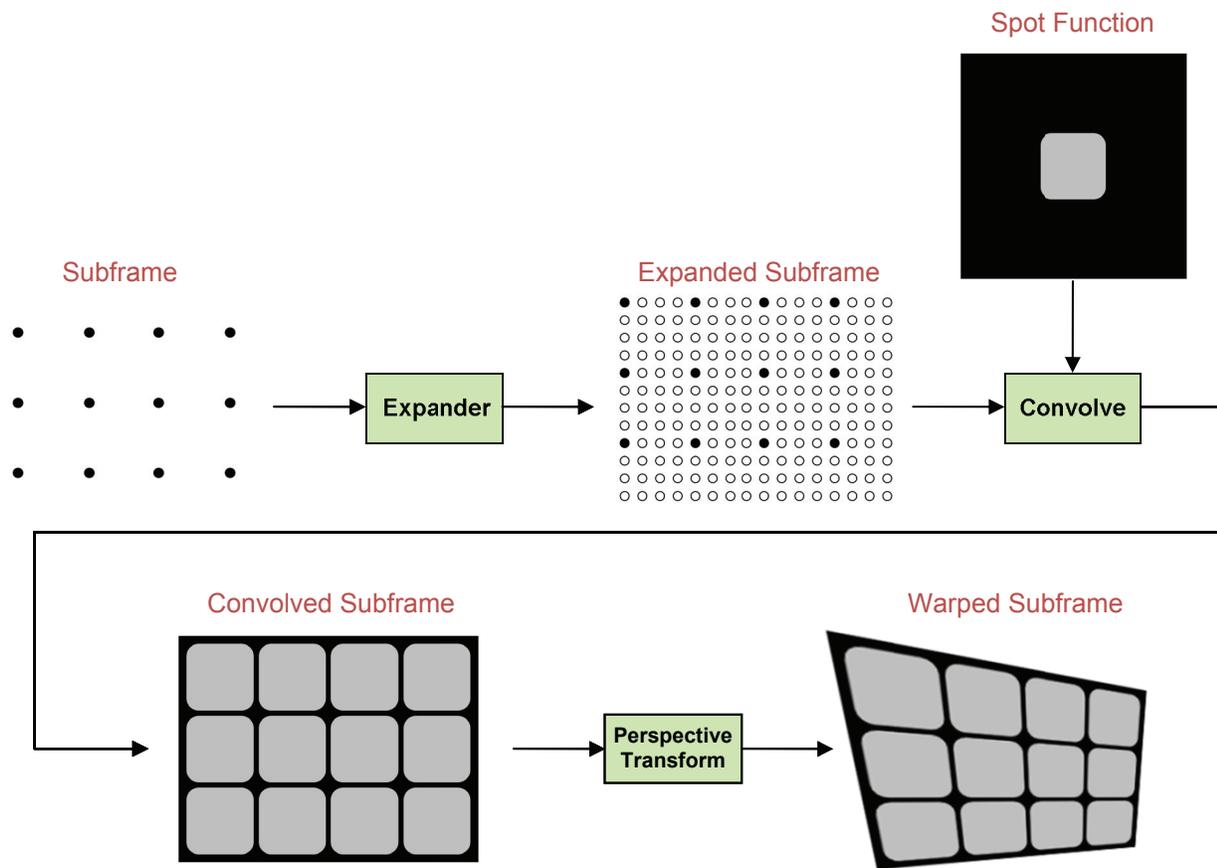
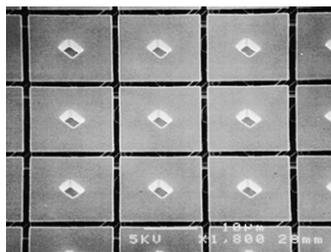


Figure 5. Graphic illustration of steps in the Renderer for one subframe.

(a) DMD Micrograph



(b) Measured Dot Function Profile

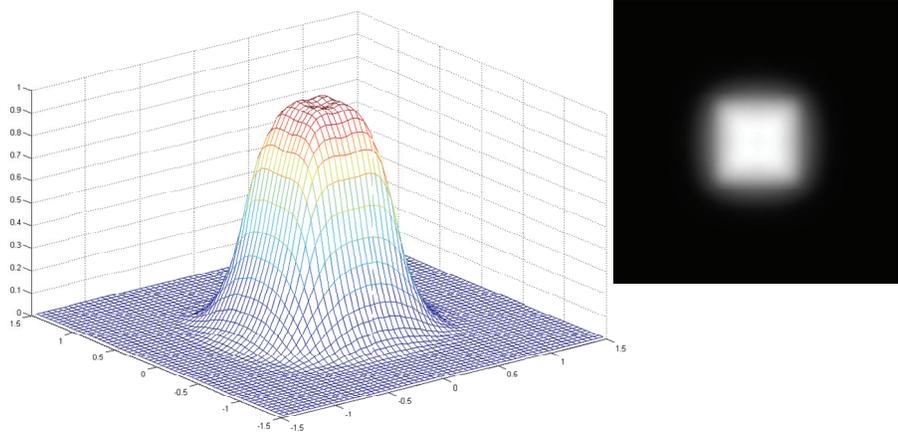
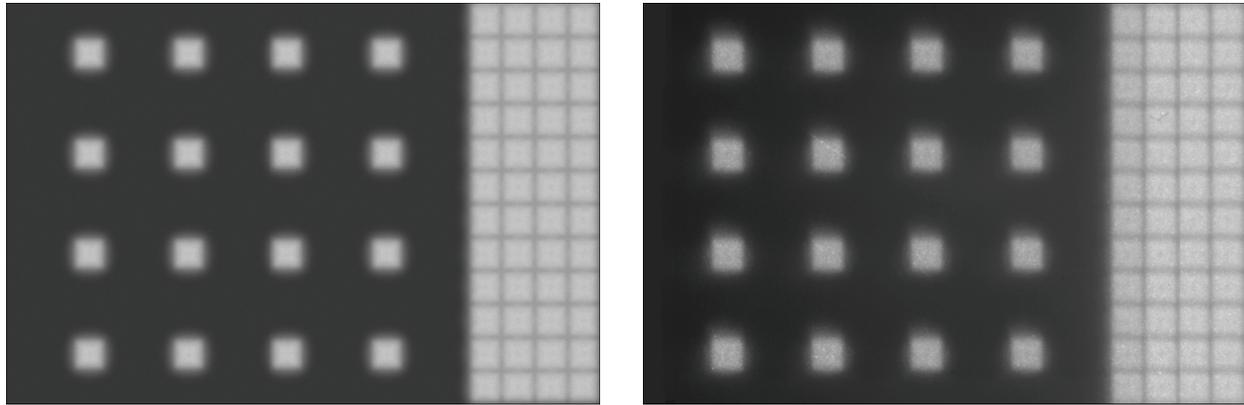


Figure 6. The DMD Dot Function



(a) Output from Simulator

(b) Macro photo of actual projection on a screen

Figure 7. Simulated and Actual output from a single projector using the averaged spot function for a bitonal test pattern.

The results are stunningly accurate. While expected in single frame projections, the persistence of the “screen door” effect in multiple overlaid projector images was thought to be a hardware/optics error before the simulator showed its presence as is evident in Figure 2.

### 3. TEST PROCEDURE

The industry accepted method<sup>6,7</sup> for assessing electronic display resolution performance uses “grill” patterns of alternating bars of black and white. The minimum and maximum output luminance ( $L$ ) orthogonal to the grill is used to compute the Michelson Contrast,  $C = (L_{\max} - L_{\min}) / (L_{\max} + L_{\min})$ . This simple approach is too narrow for our unique systems that are space-varying and generate detail finer than a subframe pixel.

The grills from our Test Pattern Generator vary in period, phase, orientation and amplitude. We analyze the output one line at a time and average in the other dimension. But instead of the direct Michelson Contrast our method uses the Fourier amplitude of the fundamental frequency corresponding to that of the grill. There are two advantages for this choice. First for space varying results it is more consistent and repeatable than using a moving average for direct measurement<sup>8</sup>. Secondly, it rewards displays that retain the squareness of the grill –the Michelson Contrast measure is identical for both sine waves and square waves of the same amplitude. To eliminate bias due to phase, each test is repeated over 4 subpixel phases and averaged.

### 4. RESULTS

An example of a grill pattern input and output for a grill period of 1.5 is shown in Figure 8. Using the units of a subframe, which is the native resolution of a single projector, a grill period of 2.0 would be one pixel black and one pixel white. So, a grill period of 1.5 is at a resolution higher than what a single projector is capable of supporting. A single projector must alias at that frequency as is evidenced in the figure, where as the 10 overlaid projector case correctly represents all the bars of the input grill.

We found that in measuring contrast modulation, results can vary considerably as a function of the phase between the grill pattern and the sampling grid. This can be seen in the plot in Figure 9 for an input grill period of 2.0. At phase = 0, the single projector case performs very well –better than all other multiple overlay cases. Because of this accident of phase, the alignment works very well but the contrast modulation measure is not representative of the real case of arbitrary phase. This is evidenced in the readings at quarter period shifts where the overlay cases perform more consistently and the single projector case degrades; in fact, at a phase of one half period the perfect misalignment of grill and sampling grid result in an output that is just a fixed gray level with an associated contrast modulation of zero.

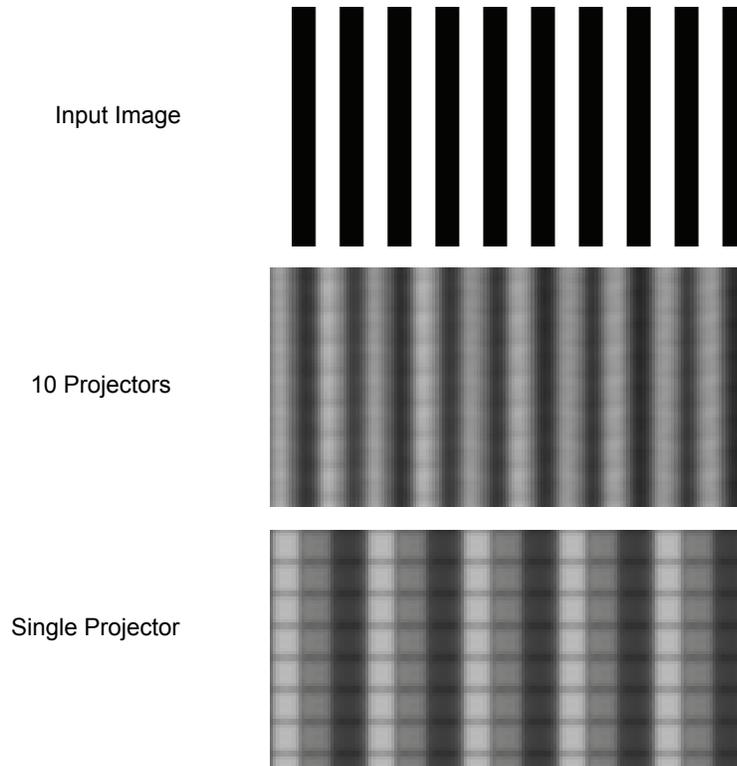


Figure 8. Example input and output for Grill Period = 1.5

As another example we show the contrast modulation results for period = 3.0 in Figure 10 where the variations due to phase are still evident, though not as extreme. Along with the single projector case we also show results for 2 and 4 position wobulation (which yield identical results), and a 10 projector system using the DMD dot function depicted in Figure 6(b). For reference we also ran the test with a smaller, idealized dot size tailored to the sampling configuration. For 4-position wobulation the “ideal” dot function used is a box function with sides of length  $1/2 \times 1/2$ , for a dot of size  $1/4$  in terms of the size of a subframe pixel. For the 10-projector case the ideal dot is of size  $1/10$ , that is, a box function with sides of length  $\sqrt{1/10}$ . For 2-position wobulation, the ideal dot function a box with sides  $\sqrt{1/2}$  but is rotated  $45^\circ$  to properly tile the plane.

The results of this study are plotted in Figure 11 for several grill periods. The data at each grill period is the average over 4 phases to account for the phase-dependent variation. The top three plots use the ideal dot function described above. While these dot sizes may be ideal for the purpose of improving resolution, because they are smaller they would result in the projection of less light. Being that the main reason for using multiple projections is to increase total lumens on the screen, engineering such dot functions would not be justified.

The lower 5 plots in Figure 11 use the real dot function where along with the 10-projector and wobulation cases we include results for 2 and 4 projectors. Relative the 1-projector case as a reference, the data clearly shows that contrast modulation is improved when properly prepared frames are overlaid. The improvement increases as the grill period gets smaller. It is interesting to note that the results for both 2 and 4 position wobulation perform almost as well as the 10-projector case. The reason for this is that the pixels of the subframes in wobulation are overlaid in a precise homogeneous arrangement, where multiple separate projectors overlay in a nonuniform way.

In either case, the used of this simulator allows this quantification of contrast modulation and documents that improvements are achieved even at subpixel resolutions.

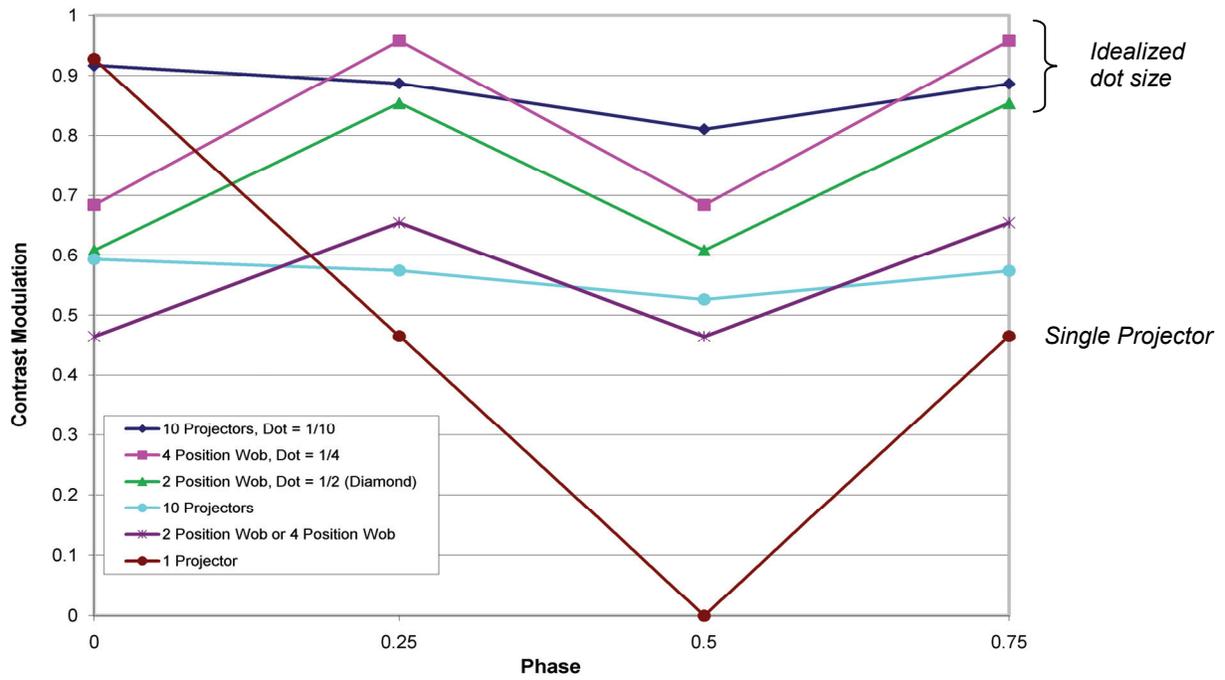


Figure 9. Contrast as a function of phase for Period = 2.0.

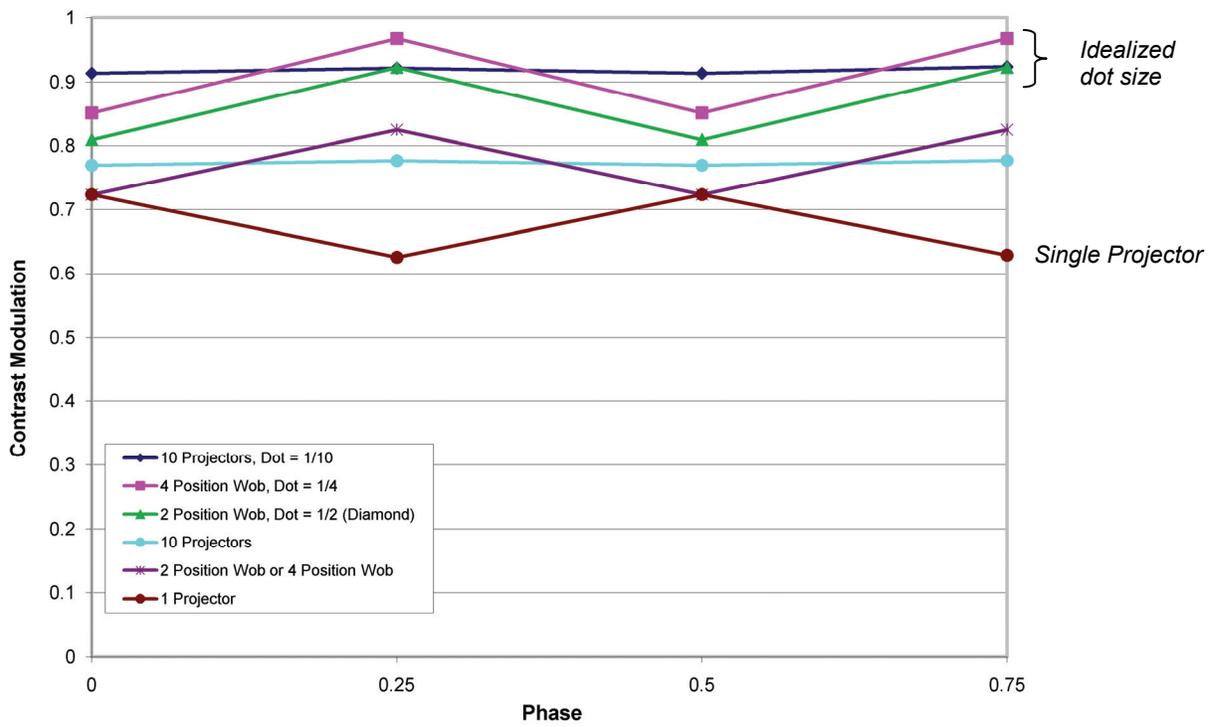


Figure 10. Contrast as a function of phase for Period = 3.0.

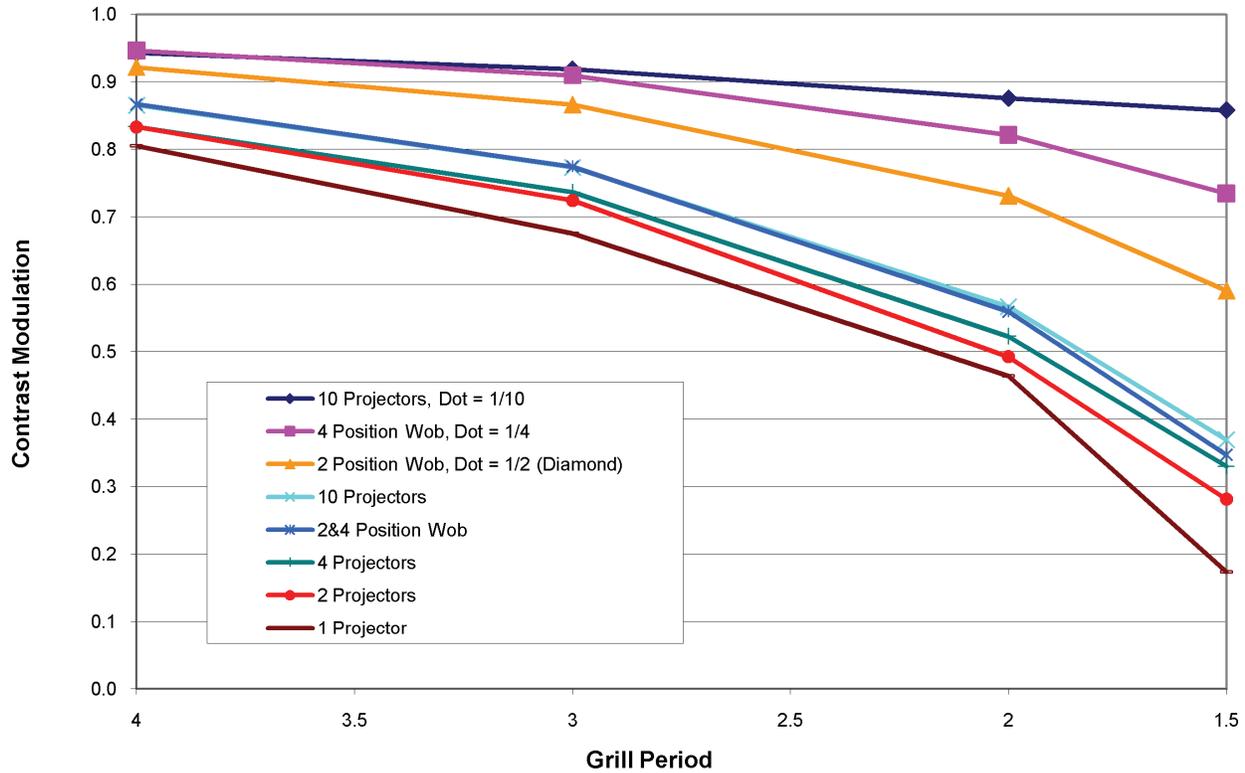


Figure 11. Contrast Modulation averaged over phase as a function of grill period.

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