Measuring the Modulation Transfer Function of Image Capture Devices: What Do the Numbers Really Mean? *

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ABSTRACT

The modulation transfer function (MTF) is a fundamental tool for assessing the performance of imaging systems. It has been applied to a range of capture and output devices, including printers and even the media itself. In this paper, we consider the problem of measuring the MTF of image capture devices. We analyze the factors that limit the MTF of a capture device. Then, we examine three different approaches to this task based, respectively, on a slant-edge target, a sinewave target, and a grill pattern. We review the mathematical relationship between the three different methods, and discuss their comparative advantages and disadvantages. Finally, we present experimental results for MTF measurement with a number of different commercially available image capture devices that are specifically designed for capture of 2D reflection or transmission copy. These include camera-based systems, flat-bed scanners, and a drum scanner.

Keywords: Modulation transfer function (MTF), slanted edge, sinewave, grill

1. INTRODUCTION

The modulation transfer function (MTF) is a fundamental tool for assessing the performance of imaging systems. Particularly, MTF is a commonly used metric for defining the spatial resolution characteristics of imaging systems. MTF is a function of spatial resolution. It has been applied to a range of capture and output devices, including printers and even the media itself. In this paper, we consider only MTF characterization of image capture devices.

Usually, we can use three kinds of targets to measure the MTF: Slanted-edge target, Sine-wave target, and grill(bar/square) pattern. Slanted-edge target is a square target contains four slanted edges which are contrast edges compared with the background. The method to measure the MTF by slanted-edge target is called slanted-edge method or slanted-edge analysis.[1] This is the least computational method for MTF measurement compared with the methods using other two targets. Also, this method is a single measurement. Instead, both Sinewave target and grill pattern need several measurements on different frequencies in order to get the MTF.

Sinewave target is a set of reflective/transmission sinewave patterns of different frequencies.[2] There are two methods to measure the MTF by this target. One is directly based on the definition of the MTF. For each frequency of the sinewave pattern, we measure the modulation of the captured image. The MTF value of a certain frequency is equal to the ratio of the modulation of the captured image to the modulation of the original image. On the other hand, we also can use a Fourier analysis on the sinewave target. The Fourier analysis is more robust to the noise due the quality of the reflective photo paper or transmission film. However, to produce a sinewave target is the most difficult. The quality of the target is very important to the accuracy of the measurement.

The third target is the grill pattern.[3] Or they can be called square pattern, or bar pattern. They are easier to generate than sinewave targets, while the ratio of the modulation of the scanned image to the modulation of the original image is defined as contrast transfer function (CTF). It is different from the MTF. There are formulas to transfer between the MTF and the CTF.[4] In addition, we also can use the Fourier analysis to calculate the modulation of each frequency. After getting the CTF, we need to use the formula to transfer the CTF to MTF. The transformation between the CTF and MTF is not a simple linear transformation, so it inevitably induces some distortion in the MTF.

The remainder of this paper is organized as follows: In Sec. 2, we briefly describe the terminology and definitions that are used in this paper, and the targets will be used. In Sec. 3, we compare the relationship among the methods based on the three targets. We discuss the advantage and disadvantage of these methods. In Sec. 4, experimental results are provided. Finally, we draw our conclusions in Sec. 5.

2. PRELIMINARIES

In this section, we will first introduce the definitions of the modulation transfer function (MTF), other transfer functions related to MTF, and some spread functions that can help with calculating MTF.

2.1 Modulation Transfer Function (MTF) and other transfer functions

In the optical and image capture system, we have the formula:

$$g(x, y) = h(x, y) * * f(x, y) \tag{1}$$

Here, (x, y) is the spatial coordinates; $f(x, y)$ is the original input image; $g(x, y)$ is the captured image; $h(x, y)$ is considered as the system's impulse function; ** denotes 2-D convolution. When the Eq. 1 is converted into the Fourier domain, we will have:

$$G(u, v) = H(u, v)F(u, v) \tag{2}$$

In Eq. 2, (u, v) is the coordinates in the frequency domain; $G(u, v)$, $F(u, v)$, and $H(u, v)$ are the Fourier transform of the captured image, original input image, and the impulse function, respectively. $H(u, v)$ is also defined as the transfer function. When $H(u, v)$ is normalized to have the unit value at zero spatial frequency, if applicable, in a classic optical or image capture system, $H(u, v)$ is referred to as the optical transfer function (OTF).[5] Usually, the OTF is a complex function, having both the magnitude and phase portion. The magnitude portion is referred to as modulation transfer function (MTF) and the phase portion is referred to as phase transfer function (PTF).

$$OTF \equiv H(u, v) = |H(u, v)| \, e^{-j\theta(u,v)} \tag{3}$$

As we clarified above, we have:

$$MTF \equiv |H(u, v)| \tag{4}$$

and

$$PTF \equiv \theta(u, v) \tag{5}$$

On the other hand, MTF is also defined as:

$$MTF(f) = \frac{M_{captured}(f)}{M_{original}(f)} \tag{6}$$

In the equation 6, $M_{captured}$ and $M_{original}$ are the modulations of the captured image and the original target. The original modulations will come with the target from the manufacturer. Modulation of the image is defined as:

$$Modulation \equiv \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{7}$$

I_{max} and I_{min} are the maximum and minimum intensities of the image, as shown in Fig. 1 [6]

$$Modulation \equiv \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2(A - B)}{2B} = \frac{A - B}{B} \tag{8}$$
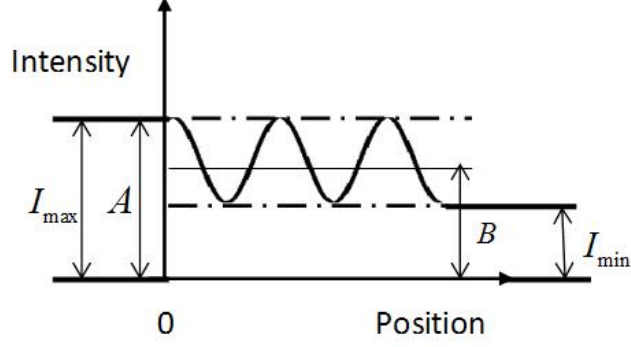
Figure 1. Modulation of the sinewave.

In the rest of the paper, we will use the definition of point spread function (PSF), line spread function (LSF) and edge spread function (ESF). When the input image is a delta function $\delta(x, y)$, we will have the captured output image as a point spread function:

$$PSF(x, y) \equiv g(x, y) = h(x, y) * *\delta(x, y) = h(x, y) \tag{9}$$

When the input image is a line like:

$$f(x, y) = \delta(x)1(y) \tag{10}$$

Then the output image $g(x, y)$ is a line spread function:

$$LSF(x) \equiv g(x, y) = h(x, y) * *f(x, y) = h(x, y) * *[\delta(x)1(y)] = PSF(x, y) * *[\delta(x)1(y)] \tag{11}$$

The y direction convolution with a constant is equivalent to an integration over the y direction, so

$$LSF(x) = \int_{-\infty}^{\infty} h(x, y') \, \mathrm{d}y' \tag{12}$$

Therefore, we can tell that the MTF of the x direction is the magnitude of the one-dimensional Fourier transform of the line spread function:

$$MTF(u) = MTF(u, 0) = |\mathcal{F}[LSF(x)]| \tag{13}$$

When the input image $f(x, y)$ is a step function,

$$f(x, y) = u(x)1(y) \tag{14}$$

Then the output image $g(x, y)$ is an edge spread function:

$$ESF(x) \equiv g(x, y) = h(x, y) * *f(x, y) = h(x, y) * *[u(x)1(y)] = PSF(x, y) * *[u(x)1(y)] \tag{15}$$

The convolution between the PSF and a constant produces an LSF in the y direction; the convolution between the PSF and a unit-step function is an integration:

$$ESF(x) = \int_{-\infty}^{x} LSF(x')\mathrm{d}x' \tag{16}$$

Therefore, the derivative of the ESF generates the LSF in the x direction:

$$\frac{\mathrm{d}}{\mathrm{d}x}ESF(x) = \frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{x} LSF(x')\mathrm{d}x' = LSF(x) \tag{17}$$

Thus, MTF of the x direction can also be calculated from the ESF:[7]

$$MTF(u) = MTF(u, 0) = |\mathcal{F}[LSF(x)]| = \left|\mathcal{F}[\frac{\mathrm{d}}{\mathrm{d}x}ESF(x)]\right| \tag{18}$$

2.2 Slanted-edge, Sinewave, and grill pattern target and corresponding measurement methods

2.2.1 Slanted-edge targets and method

The slanted-edge targets are shown in Fig. 2: QA-61 and QA-62 [†].
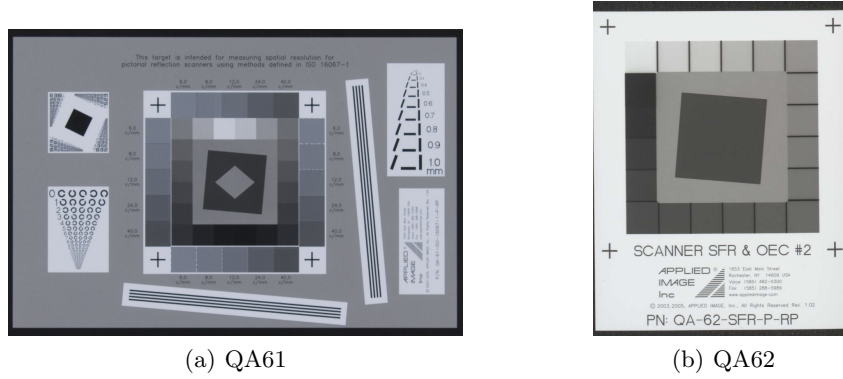

(a) QA61 (b) QA62

Figure 2. Slanted edge targets.

The basic idea for this method is that after getting the LSF by derivative of the ESF, compute the Fourier transform of the LSF. Normalize the Fourier transform value in order to get the spatial frequency response (SFR), denoted as the MTF.

2.2.2 Sine-wave target and methods

The sinewave target is Sine M-13-60-RM [†] which is shown in Fig. 3. Each of the sinewave patterns has a different
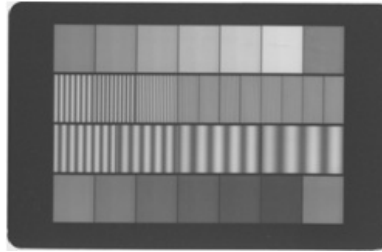

Figure 3. Sinewave target.

frequency. There are two methods to calculate the MTF by the sinewave target. One is the direct method. Using Eq. 6 and Eq. 24, we can determine the MTF of all the frequencies supplied with the sinewave. We can then plot a MTF value versus the frequency. The other method is the Fourier method.[8] The modulation of the captured image can be calculated by

$$Modulation(f) = \frac{Fundemental\ frequency\ component}{DC\ component} \tag{19}$$

2.2.3 Grill target and methods

The grill pattern is T-60 [†] which is shown in Fig. 4.

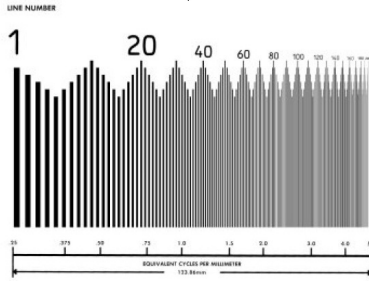[†]   Applied Image Inc., 1653 East Main Street, Rochester, NY 14609 USA

Figure 4. Grill patterns.

3. MTF MEASUREMENTS USING THREE TARGETS

3.1 Method using slanted-edge targets

The flowchart of running 'SFRedge' ‡ software is shown in Fig. 5. We first choose the region of interest: the image between the crosses. After selecting the Region of Interest, we can choose regions marked with yellow
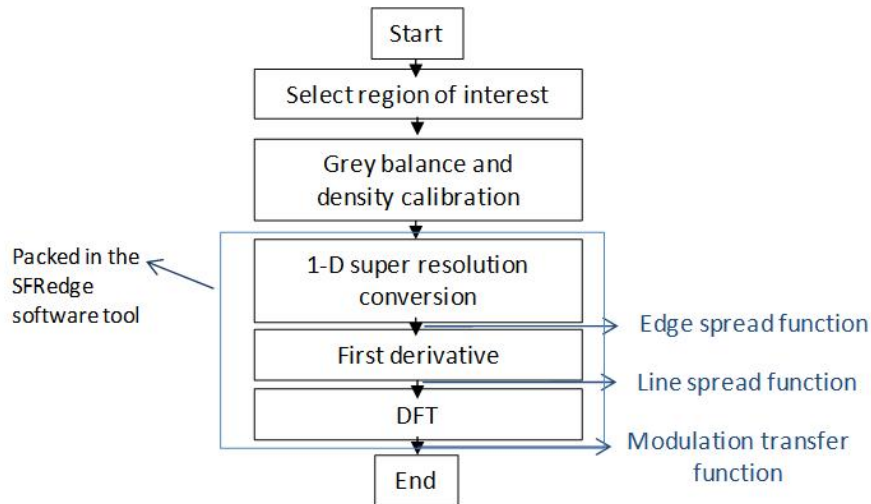

Figure 5. Flow chart of the slanted-edge method.

rectangles contain the slanted edges used to compute the SFR. The regions marked with the red rectangles are the regions used for density calibration.

3.2 Method using sine-wave targets

Here, we take as an example the use of a reflective sinewave target printed on photo paper to measure the MTF of a scanner. The flowchart of the direct method is shown in Fig. 8. When getting the scanned sinewave image, we calculate the averaged peak value and the mean value (direct component) of the sinewave. Two auxiliary steps to be completed before we compute the modulation: gray balance of the scanner and density calibration of the reflective target. The modulation is based on the optical density, using Eq. 24.

The flowchart of the Fourier analysis method is shown in Fig. 9. The difference between the Fourier method and the direct method is that after doing the Fourier analysis, we utilize Eq. 19 to compute the modulation. We determine that the DC component energy $F(0)$ and the energy $F(N)$ at N, the number of sinusoidal cycles selected in the 'Region of Interest'. Fourier analysis has its advantage over the direct method because by Fourier

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‡ Image Science Associates, http://www.imagescienceassociates.com
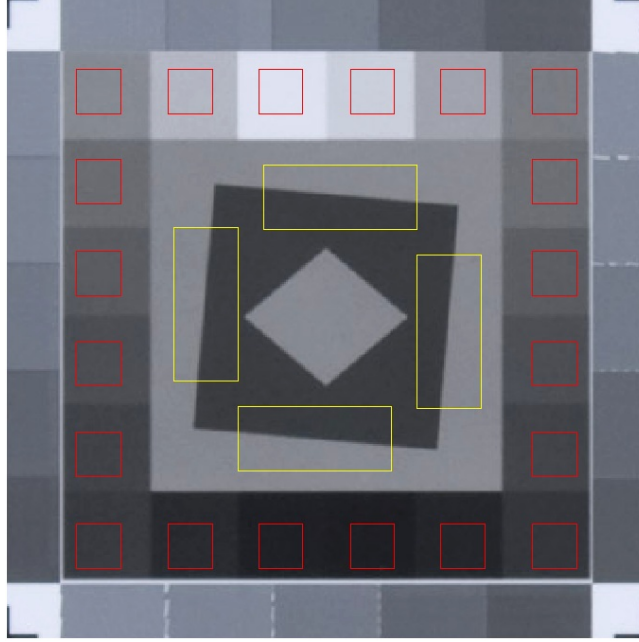
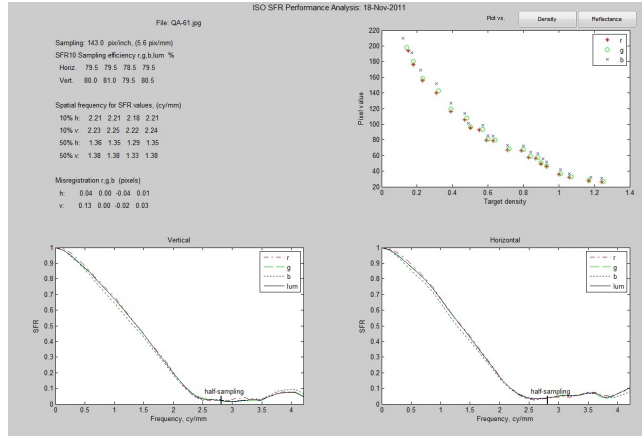Figure 6. Region of Interest of the slanted-edge method.

Figure 7. Result output of the SFRedge.

transformation, we can effectively suppress the random white noise. To prove the Fourier analysis method to calcuate the MTF, we can first assume that the sinewave target we have only contains the DC component and the fundamental component:

$$x(t) = \beta + \alpha \cos(2\pi f_0 t) \tag{20}$$

$$= \beta + \frac{\alpha}{2} e^{j2\pi f_0 t} + \frac{\alpha}{2} e^{-j2\pi f_0 t} \tag{21}$$

From Eq. 21, β is the DC component B, and α is the twice of the fundamental frequency. Also, from Fig. 1, we can discern the relationship among α, β, A (maximum density of the sinewave), and B (the DC component of the sinewave).

$$A = \alpha + \beta \tag{22}$$
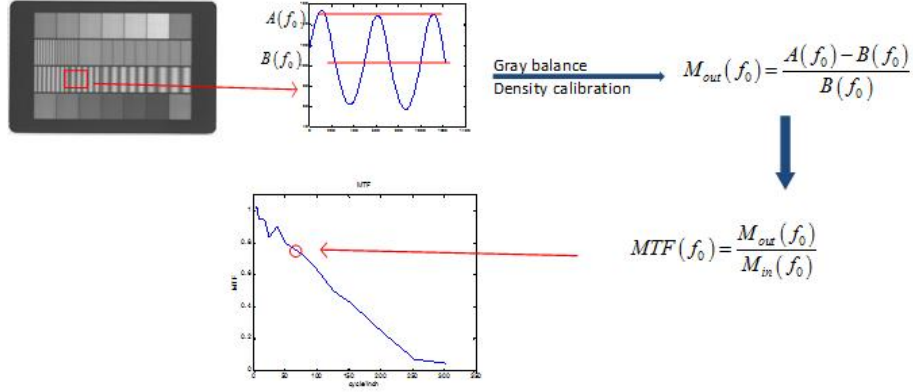
and

$$B = \beta \tag{23}$$

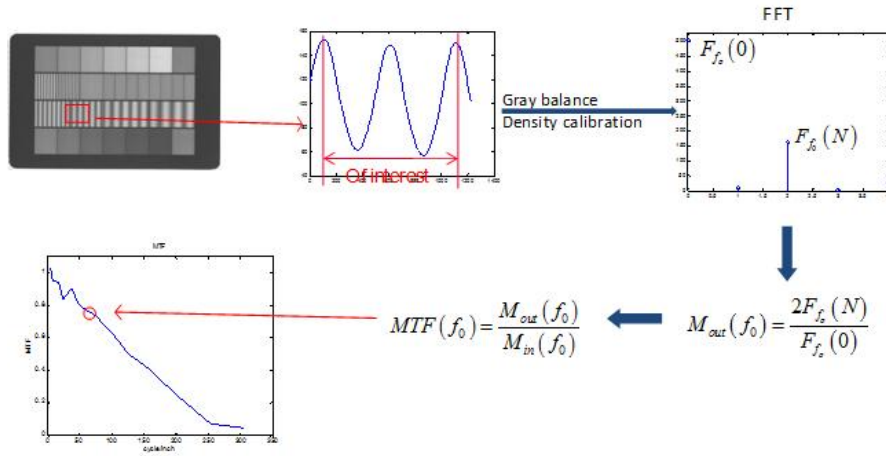Figure 8. Flow chart of the direct method using sinewave target.

Figure 9. Flow chart of the Fourier analysis method using sinewave target.

Substituting Eqs. 22 and 23 into the Eq. 24, we can get:

$$Modulation = \frac{A - B}{B} = \frac{\alpha}{\beta} \tag{24}$$

Because we select several cycles of the sinewave, in the frequency domain, it has an upsampling effect, which means that the fundamental frequency energy is located at the number of the sinewave cycles. Therefore,

$$Modulation = \frac{\alpha}{\beta} = \frac{2F(N)}{F(0)} \tag{25}$$

3.3 Method using grill pattern

An accurate sinewave target is very difficult to fabricate if we want only the single frequency in the sinusoidal function. Therefore, sometimes we use the grill/bar/square wave target, which is much easier to produce. However, we similarly use the direct method for sinwave target, the ratio of the captured modulation and the original modulation is larger than the MTF, because the bar pattern has lots of frequencies other than the fundamental frequency. It will include the energy we do not need. We defined, for an infinite square wave, the contrast transfer function (CTF) as the ratio of the captured and original modulations, which is similar the MTF defined for sinewave target. According to the Coltman formulas[4] given by Eqs. 26 and 27,

$$MTF(f) = \frac{\pi}{4}\left[CTF(f) + \frac{1}{3}CTF(3f) - \frac{1}{5}CTF(5f) + \frac{1}{7}CTF(7f) + \frac{1}{11}CTF(11f) - \frac{1}{13}CTF(13f) + \cdots\right] \tag{26}$$

and

$$CTF(f) = \frac{4}{\pi}\left[MTF(f) - \frac{1}{3}MTF(3f) + \frac{1}{5}MTF(5f) - \frac{1}{7}MTF(7f) + \frac{1}{9}MTF(9f) - \frac{1}{11}MTF(11f) + \cdots\right] \quad (27)$$

Considering the Coltman formulas, in order to calculate the MTF, we need a set of higher harmonically related frequencies for CTF measurement. Nevertheless, it is impractical to have many harmonic frequencies. Therefore, we usually take the first term in the Eq. 27 as an approximate MTF.

$$MTF(f) \simeq \frac{\pi}{4}CTF(f) \quad (28)$$

As Eq. 28 shows, usually the CTF is higher than the according MTF. If we directly take the measured CTF as the MTF, instead of multiplying the CTF by $\frac{\pi}{4}$, we get a high-biased MTF.

Like the sinewave target, grill pattern also has two methods: direct method and Fourier method. The direct method for the grill pattern has one more step than the sinewave target, because the ratio of the modulations is the CTF. To convert to the MTF, we need to multiply the CTF by $\frac{\pi}{4}$. The Fourier method of the grill patterns has two differences compared with the method of the sinewave target. One is like the direct method. We need to convert the CTF into MTF by multiplying $\frac{\pi}{4}$. The other difference is that the modulation by the Fourier method needs to be normalized by $\frac{4}{\pi}$:

$$Modulation = \frac{2F(N)}{F(0)}\bigg/\left(\frac{4}{\pi}\right) \quad (29)$$

For example, we compare the modulations and MTF/CTF from the sinwave target and square target:

	Sinewave	Sqaure wave
DC component	1	1
Amplitude	0.5	0.5
Number of cycles	4	4
Modulation	0.5	0.5
Normalized modulation by Fourier method	$\begin{aligned}\frac{2F(N)}{F(0)} &= \frac{2\times 200}{800}\\ &= 0.5\end{aligned}$	$\begin{aligned}\frac{2F(N)}{F(0)}\bigg/\left(\frac{4}{\pi}\right) &= \frac{2\times 254.6584}{800}\bigg/\left(\frac{4}{\pi}\right)\\ &= 0.5\end{aligned}$
MTF or CTF (Assuming the input modulation is 1)	$MTF = \frac{0.5}{1} = 0.5$	$\begin{aligned}CFT &= \frac{0.5}{1} = 0.5\\ MTF &= \frac{\pi}{4}CFT = \frac{\pi}{8}\end{aligned}$

4. EXPERIMENTAL RESULTS

We used our slanted-edge, and sinewave targets on the scanner and high-resolution cameras, respectively. The method used for the sinewave target is the Fourier method. We made measurements on a high-resolution scanner A. The addressability is up to 2400 dpi. Camera B with higher addressability up to around 7600 dpi is also measured. Fig. 10 shows the MTF measurement on scanner A. The slanted edge measured is perpendicular to the scanning direction. And the direction along with the varied frequency on the sinewave target is parallel to the scan image.

Fig. 11 and Fig. 12 show the MTF of two perpendicular directions on the scanner A. Fig. 11 uses the slanted-edge target; Fig. 12 uses the sinewave target. We can observe that the MTF values begin to diverge when the frequency increases. Therefore, the MTF of scanner A is not direction independent.
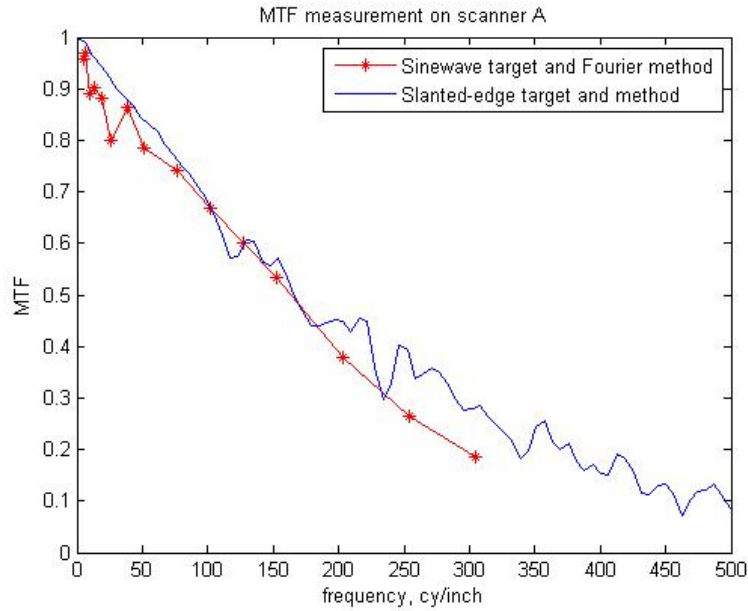
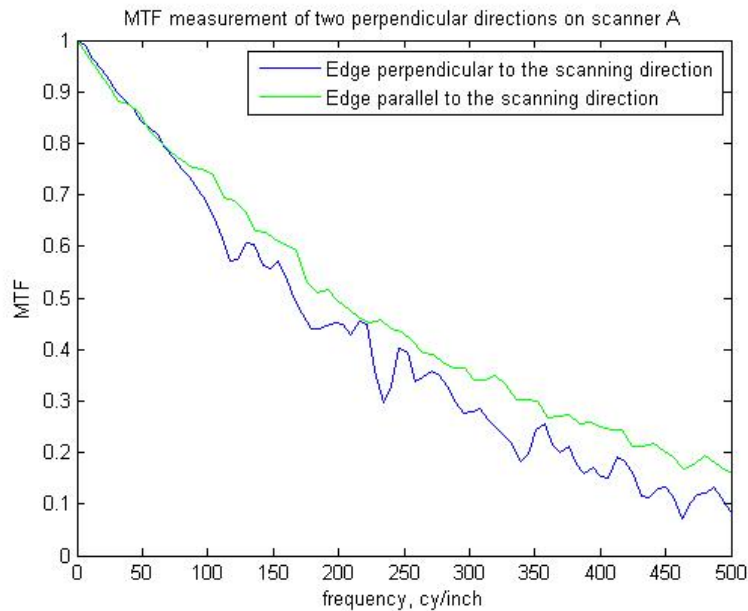Figure 10. MTF Measurement on scanner A.

Figure 11. MTF Measurement of two directions on scanner A, based on slanted edge method.

Fig. 13 is the MTF measurement for the camera B. Two targets and methods are compared here. One is the sinewave target and direct method; the other is the slanted-edge target and method.

5. CONCLUSION

In this paper, we discuss the three different targets based on two different but related definitions of the MTF. We illustrate the mathematical theory behind the three targets and methods. Experiments with the targets are conducted. The relationship among the targets are explored.
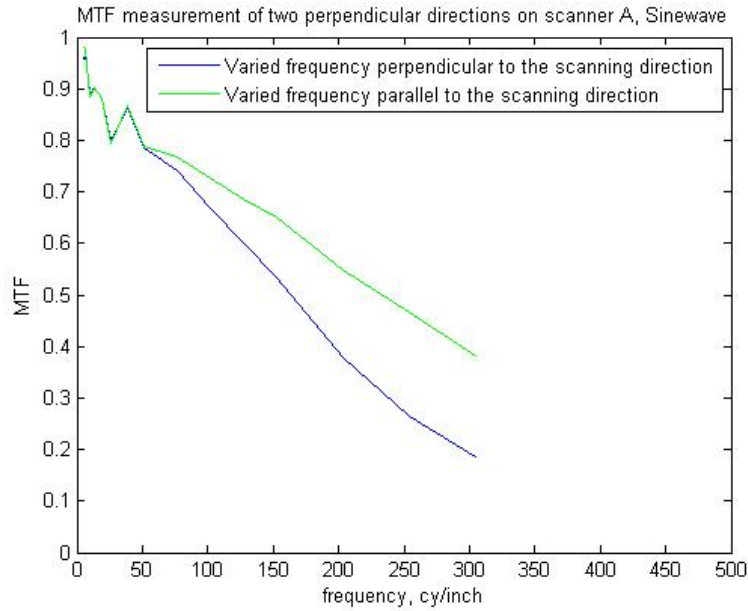
Figure 12. MTF Measurement of two directions on scanner A, based on sinewave method.
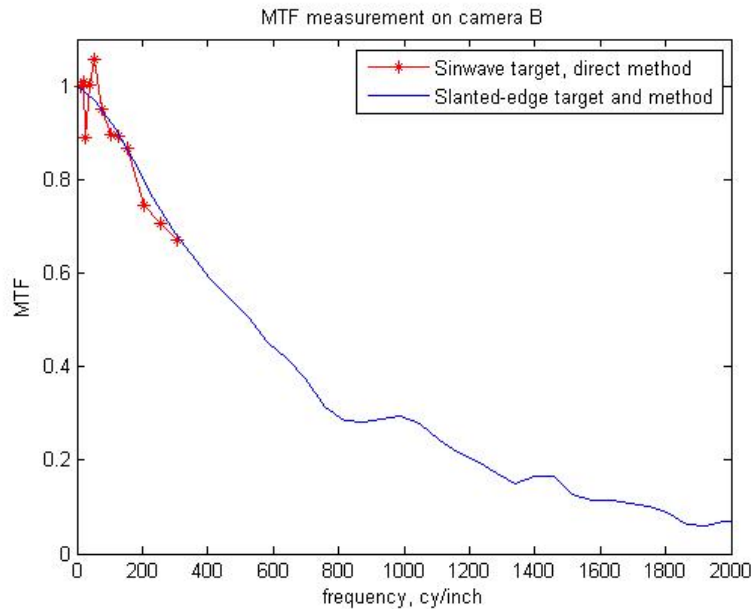

Figure 13. MTF Measurement of camera B based on Sinewave and Slanted-edge methods.

REFERENCES

[1] Burns, P., *SFR-Edge Installation and Users Guide*, 1 ed. (Oct. 2008).

[2] Applied Image Inc., 1653 East Main Street, Rochester, NY 14609 USA, *Sinusoidal Array M-13-60-S-RM*.

[3] Fiske, T. G. and Silverstein, L. D., "Display modulation by fourier transform: a preferred method," *Journal of the Society for Information Display* **14**, 101–105 (January 2006).

[4] Nill, N. B., *Conversion Between Sine Wave and Square Wave Spatial Frequency Response of an Imaging System*. The MITRE Corporation.

[5] Boreman, G. D., [*Modulation Transfer Function in Optical and Electro-Optical Systems*], SPIE.

[6] Lamberts, R. L., *Use of Sinusoi Test Patterns for MTF Evaluation.*

[7] "Iso12233: Photography – electronic still-picture cameras – resolution measurements."

[8] Jang, W. and Allebach, J. P., "Characterization of printer mtf," *Journal of Imaging Science and Technology* **50** (May/June 2006).